International Journal of Theoretical Physics, Vol. 30, No. 4, 1991

Lobachevsky Geometry and Unsolved Problems of Solar Cosmogony

A. S. Rabinowitch[1]

Received June 4, 1990

An axiomatic method of constructing physics in Lobachevsky space is proposed. Dynamical equations are applied to explain the principal properties of the solar system, its planets, their satellites, and rings.

1. PRESENT SITUATION IN SOLAR COSMOGONY

It is well known that despite numerous attempts to explain the origin of the solar system (beginning with Descartes' hypothesis in 1644), to this day there is no theory making it possible to interpret the main peculiarities of the planets of the solar family. A number of once popular theories managed to give good explanations for just some of these peculiarities, but insurmountable difficulties arose for the rest.

Let us give here only the most interesting properties of the solar system (Klimishin, 1980):

1. The orbits of the planets lie practically in the plane of the solar equator and are close to circular.

2. Most of the secondary planets move in almost circular orbits in the plane of their equator.

3. The principal planets (Saturn, Jupiter, Uranus, and, according to the latest data, Neptune) have ring systems in their equatorial planes.

4. All the planets move about the sun in the same direction, coinciding with that of the proper rotation of the sun.

5. 99.8% of the whole solar system mass comes from the sun and only 0.2% the planets, whereas the planets have 98% of the angular momentum.

[1]Bryanskaya Ulitza 12-63, Moscow 121059, USSR.

0020-7748/91/0400-0521$06.50/0

The conceptions of the genesis of the solar system based on Newtonian mechanics and the Euclidean geometry of space have the following difficulties.

Only three scenarios of incipient planetary formation are possible:

1. Formation from the same gaseous dust cloud as the sun (Kant).
2. Capture by the sun of the planetary cloud (Schmidt).
3. Separation of the planets from the sun in the process of evolution (Laplace, Jeans, and others).

While the first and the third variants meet great difficulties in the explanation of the above property 5 of the solar system (contradiction of the law of conservation of angular momentum), the second variant does so with the explanation of property 1 (captured planets have no necessity to move in the plane of the solar equator).

The properties 2–4 are no easier to explain. General relativity when brought into these problems cannot dramatically change the situation, since the corrections it could being would be small for the lifetime of the solar system (Landau and Lifshitz, 1973).

That is why to solve the question of the genesis of the solar system new approaches are needed. As one of these, let us turn to Lobachevsky geometry, which is the most natural generalization of Euclidean geometry.

As was demonstrated by Friedman (1979), the cosmological equations of general relativity (with the cosmological term) allow a stationary Lobachevsky geometry if, besides common matter, there exists matter with a negative density of energy. Such matter can be the physical vacuum viewed in the Dirac sense (Sokolov *et al.*, 1979).

Let us take up the task of building physics in the stationary Lobachevsky space. Since this can be realized only in the presence of a physical vacuum, it is necessary to take into consideration the influence of the latter on the bodies moving relative to it.

The question of such influence is open, and so to build up physics in Lobachevsky space we shall take the axiomatic approach—we shall add some natural physical axioms to the Lobachevsky geometry.

But first let us turn to some characteristics of Lobachevsky geometry important for our purpose.

2. SOME DATA FROM LOBACHEVSKY GEOMETRY

In Lobachevsky planimetry there is an infinity of straight lines passing through a point beyond a given one without crossing it. The two lines bordering a line and coming unlimitingly near it at infinity on the left and one the right were named by Lobachevsky the left and the right parallel lines, respectively.

Consider the coordinate system made by three mutually perpendicular straight lines of Lobachevsky space that have a common point O. This coordinate system we shall call Cartesian and the three orthogonal projections of any point down these straight lines (axes of the system) counting from point O we shall call its Cartesian coordinates ξ_1, ξ_2, ξ_3.

Along with this we shall consider Beltrami coordinates

$$y_i = \mathrm{th}(\xi_i/R) \tag{1}$$

where R is the radius of the Lobachevsky space.

The coordinates y_i fill the inner space of a Euclidean sphere with radius equal to one (Yefimov, 1978):

$$y_1^2 + y_2^2 + y_3^2 < 1 \tag{2}$$

Movement is an important notion in Lobachevsky geometry as in any other. Transfigurations y_i transform the sphere (2) into itself and are linear fractional (Yefimov, 1978; Alexeevsky *et al.*, 1988):

$$y_i' = \left(\sum_{j=1}^{3} a_{ij} y_j + b_i \right) \Bigg/ \left(\sum_{j=1}^{3} \alpha_j y_j + \beta \right) \tag{3}$$

where a_{ij}, α_j, b_i, and β are constants, and y_i' are the transformed coordinates y_i.

Consider also parabolic coordinates x_i:

$$x_i = Ry_i/(1 - y_1) \tag{4}$$

where the inside of the sphere will form the inside of the paraboloid.

These coordinates are convenient when examining a set of straight lines parallel to the abscissa of a Cartesian coordinate system in the positive direction.

As can be easily seen, movements that change the set of straight lines into themselves are described by the following linear transformations x_i' and x_i'' of coordinates x_i:

$$\begin{gathered} x_1' = \lambda x_1 + \sqrt{\lambda}\, \alpha x_2 + \sqrt{\lambda}\, \beta x_3 + 1/2\, (\alpha^2 + \beta^2) R - R/2\, (1 - \lambda), \\ x_2' = \sqrt{\lambda}\, x_2 + \alpha R, \qquad x_3' = \sqrt{\lambda}\, x_3 + \beta R, \\ x_1'' = x_1', \qquad x_2'' = x_2' \cos\theta - x_3' \sin\theta, \qquad x_3'' = x_2' \sin\theta + x_3' \cos\theta \end{gathered} \tag{5}$$

where α, β, λ, and θ are arbitrary constants, $\lambda > 0$.

Let us make one more point we will need: in coordinates y_i and x_i the planes in Lobachevsky geometry are described by linear equations of the type

$$\sum_{i=1}^{3} \alpha_i y_i = \alpha_0, \qquad \sum_{i=1}^{3} \beta_i x_i = \beta_0 \tag{6}$$

where α_i, β_i are constants.

3. MOTION OF A BODY IN LOBACHEVSKY GEOMETRY

First let us find the equation of translation of a solid body with inertia. Let us start with some necessary axioms:

1. The points of the body should move in a straight line, the line segments should remain straight and its size should be limited.
2. The equation of motion should be covariant with respect to a change of zero time and the reference point of the Cartesian coordinate system.
3. The speed of points of the body cannot exceed the light speed.
4. When $R \to \infty$, the sought for equation should become an equation of classical mechanics.

According to the first requirement, the points of the body in question move by parallel lines (in the direction of movement), or else the body size could become unlimited.

Let us choose a Cartesian system with the abscissa parallel to these straight lines in the same direction. In this coordinate system the equation of free translation of points of the body will be

$$dx_i/dt = u_i, \qquad i = 1, 2, 3 \tag{7}$$

where

$$x_i = R\frac{\operatorname{th}(\xi_i/R)}{1-\operatorname{th}(\xi_1/R)}, \qquad t = \frac{R}{2c}\left[\exp\left(\frac{2c\tau}{R}\right) - 1\right] \tag{8}$$

ξ_i are orthogonal projections to the axes of the considered Cartesian coordinate system, τ is time, c is the light speed, and u_i are constants, the same for all points of the body, $u_2 = u_3 = 0$, $u_1 \geq 0$.

Let us demonstrate thus.

As $u_2 = u_3 = 0$, then from (7) and (8) it follows that the points of the body move along straight and parallel lines ($x_2 = \text{const}$, $x_3 = \text{const}$; when $\xi_1 \to \infty$ we have $\xi_2 \to 0$, $\xi_3 \to 0$). One can easily see that the size of the body is also limited.

The difference $x_i(t_2) - x_i(t_1)$ does not depend on the point, as is seen from (7). From this we have that the straight segments of the body [described in coordinates x_i with a couple of linear equations (6)] will remain straight under the movement.

So the first condition is guaranteed.

From (7), (8), and (5) it follows that the second is also correct.

Since, when $R \to \infty$, $x_i = \xi_i$, $t = \tau$, the fourth holds as well.

Let us denote by v the velocity of the point at the moment τ relative to the local Cartesian coordinate system chosen about it, and when $\tau = 0$ as v_0. Then from (7) and (8) we have

$$(c/v - 1) = (c/v_0 - 1)\exp(-2c\tau/R) \tag{9}$$

When $v_0 < c$ the speed v increases and $v \to c$ when $\tau \to \infty$. The extreme case is $v_0 = c$ as the velocity c becomes constant and equal to c.

With this, equation (7) is the sought for equation of free translation of the solid body.

From (9) we get that a free mass point in Lobachevsky space is subject (on the side of the physical vacuum) to the accelerating force $\mathbf{F}_L$:

$$\frac{d(m\mathbf{v})}{dt} = \mathbf{F}_L, \qquad \mathbf{F}_L = \frac{2c}{R(1+v/c)} m\mathbf{v} \tag{10}$$

where $\mathbf{v}$ is the velocity vector of the mass point relative to the local Cartesian coordinate system chosen about it, $v = |\mathbf{v}|$, and m is its relativistic mass.

4. THE MOTION OF PHOTONS IN LOBACHEVSKY SPACE

Suppose a photon is emitted toward an observer from a far enough point M of the straight line l with a momentum directed along the same line.

Let us choose a Cartesian coordinate system in which the abscissa is parallel to l and converges with it at the point M.

We shall suppose the following axiomatic requirements for the phase of the wave function of the photon:

1. The form of the phase of the wave function should be covariant with respect to the zero of time and the reference point of the coordinate system.

2. The wave, in accord with quantum mechanics, should be flat and its front should expand with the light speed c.

3. When $R \to \infty$, it should come to the classical form.

The phase φ that satisfies all these requirements has the form

$$\varphi = \omega t + \sum_{j=1}^{3} \theta_j x_j + \theta_0 \tag{11}$$

$$t = (R/2c)[1 - \exp(-2c\tau/R)] \tag{12}$$

where x_j are parabolic coordinates in the given coordinate system, and ω, θ_j are constants.

The truth of the first and the third requirements and the flatness of the wave follow easily from (11), (12), and the characteristics of the parabolic coordinates.

Let us choose the straight line l as the abscissa.

Then, owing to the axial (l) symmetry in the phase φ, we have $\theta_2 = \theta_3 = 0$ and when the abscissa is $\xi_1 = c(\tau - \tau_0)$, where $\tau_0 = (R/2c)\ln(\omega/c\theta_1)$ the wave phase will be constant. It follows that the wave would spread with the light speed.

Hence, the phase φ is what was sought.

From (11) and (12) we find that the photon frequency ν is determined by

$$\nu = \nu_0 \exp(-2\xi/R) \tag{13}$$

where ξ is the distance gone by the photon by the time τ, and ν_0 is its initial frequency when $\tau = 0$.

From (13) when $\xi \ll R$ we have Hubble's law (the red shift of photon frequency) with Hubble's constant $H = 2c/R$. In this way in Lobachevsky space the red shift in the spectra of galaxies could be explained without presuming their dispersion (and so could be interpreted as the result of a physical vacuum influence). Having in view the exponential character of the diminution of the photon frequency (13), the photometric paradox of Sheso and Olbers (Klimishin, 1983) is not difficult to explain.

5. TWO PROBLEMS OF NEWTONIAN MECHANICS

5.1. Kepler's Problem of Movement of a Mass Point of mass *m* in a Gravitational Field of Spherical Mass *M*

Taking into account the formula (10), we find for a small force $\mathbf{F}_L$ in Lobachevsky space that the dynamics of a mass point are determined by

$$m\mathbf{w} = \mathbf{F} + \mathbf{F}_L, \qquad \mathbf{F}_L = \frac{2c}{R} m\mathbf{v} \tag{14}$$

where $\mathbf{F}$ is an external force, m is the mass, $\mathbf{w}$ is the acceleration of the mass point, and $\mathbf{v}$ is its speed (we take $|\mathbf{v}| \ll c$).

Since $\mathbf{F}_L$ is small, to solve equation (14) in Kepler's problem it is convenient to use the theory of disturbances. A nondisturbed solution ($\mathbf{F}_L = 0$) is a Kepler motion in an ellipse with large semiaxis a and eccentricity e.

By the theory of disturbances (Duboshin, 1963) we have the equations

$$\frac{dp}{d\tau} = \frac{4c}{R} p \qquad \frac{de}{d\tau} = \frac{4c}{R}(\cos\varphi + e) \qquad \frac{(\gamma M)^{1/2}}{p^{3/2}} d\tau = \frac{d\varphi}{(1 + e\cos\varphi)^2} \tag{15}$$

where $p = a(1 - e)$; a and e are slowly changing parameters of an elliptic orbit, γ is the constant of gravitation, and φ is the true anomaly.

From (15), we obtain for the averages per turn of the large semiaxis a and eccentricity e, when $a \ll R$,

$$a = a_0 \exp(4c\tau/R), \qquad e = e_0 \tag{16}$$

where $a_0 = \text{const}$, $e_0 = \text{const}$.

Therefore the mass point would turn around the mass M in an elliptical spiral with a fixed eccentricity of orbits of slowly growing size.

5.2. Free Rotation of the Body about its Axis

Let us make use of equation (14) for every mass point of the body, where **F** in this case is a summary force acting upon the point by all the others. Then, taking the moment of the vectors on the right and left parts of equation (14) with respect to the axis of rotation and carrying out the sum over all points of the body, bearing in mind that the inner force moment would cancel, we easily obtain

$$d\mathbf{G}/d\tau = (2c/R)\mathbf{G}, \qquad \mathbf{G} = \mathbf{G}_0 \exp(2c\tau/R) \tag{17}$$

where $\mathbf{G}$ is the vector of angular momentum of the body and $\mathbf{G}_0$ is that when $\tau = 0$.

Hence, the angular momentum of a freely rotating body would slowly grow.

6. APPLICATIONS TO SOLAR COSMOGONY

In the introduction some basic properties of the solar system were formulated which could not be explained from the point of view of mechanics in Euclidean space. Let us demonstrate how they can be explained in the mechanics of Lobachevsky space just given.

Consider a symmetric cosmic body rotating about its axis. From (17) it follows that as the time τ increases, the angular momentum vector grows and at some instant the mass points on its equator reach the critical velocity when gravity is balanced by the centrifugal force. At the same instant these mass points draw themselves away from the body and start rotating about it along the equatorial circumference. From (16) it follows that the detached mass points would move by in a circular spiral. The radius of a circuit as the time goes on will slowly grow.

Within the Rosh radius [which characterizes the maximum distance from the center of the body when large enough objects cannot yet exist due to the tidal raising forces that break them (Klimishin, 1980)], the detached mass points would form a ring around this cosmic body. When at last all these mass points go beyond the Rosh radius, then they can condense into a secondary planet of the cosmic body. By the central symmetry of the ring, the circuit of the spiral orbit will be circular [it is worth noting that the planetary rings are really situated inside the Rosh radius and the secondary planets (moons) outside this radius].

So the formulas (16) and (17) allow us to draw the following conclusions:

1. After a large while rings are created about a rotating celestial body in its equatorial plane, and then moons are formed from these rings when they go beyond the Rosh radius.

2. All these moons of a celestial body will move about it in its equatorial plane in a circular spiral. With the flow of time its radius will grow.

These conclusions can account for the main characteristics of the solar system: movement of the planets in almost circular orbits practically in the solar equatorial plane in the direction of rotation of the sun itself, a similar property of most of their moons, disposition of their rings in the equatorial plane, and the great value of the angular momentum of the planets.

It should be mentioned that the ejection of matter from a rotating body when its velocity because of (17) reaches the critical value on the equator ends when the velocity of equatorial points drops significantly, i.e., as a result of an explosion. The reason for the explosion would be the breach of equilibrium of the body resulting from the breaking away of a sufficiently large amount of matter from the equator. The same explanation could fit the observed explosions of stars, which also has not found any satisfying explanation (Gorbatzky, 1979).

We should note that, considering (16), the stars in galaxies would follow spirals, which gives us a way to interpret the origin of the spiral structure of galaxies.

As an application of formula (9), let us turn our attention to the enigmatic fact of the fantastic values of energy reached by particles of cosmic rays. This phenomenon can be explained by noting that, in accordance with (9), the velocity of particles when the time is very long can become extremely near to the light speed.

Let us now consider the calculation of the space radius R and Hubble's constant H by the formula given above:

$$H = 2c/R \tag{18}$$

To this end, let us regard the movement of the Moon about the Earth. Considering (16) averaged per turn, the angular velocity ω of revolution is

$$\omega = (\gamma M/r_0^3)^{1/2} \exp(-6c\tau/R) \tag{19}$$

where r_0 is the average distance of the Moon from the Earth at $\tau = 0$, M is the Earth's mass, and γ is the constant of gravitation.

From (18) and (19) it easily ensues that the angular acceleration (in this instance, deceleration) of the Moon ε is

$$\varepsilon = -6c\omega/R = -3H\omega \tag{20}$$

On the other hand, it is known that the average longitude of the Moon really suffers deceleration, and the correction $\Delta\lambda$ for this slowing down is (Abalakin, 1979)

$$\Delta\lambda = -11''.22\tau_c^2 \tag{21}$$

where τ_c is the time in Julian centuries.

Substituting for (20) the known value of ω for the Moon and the value of ε from (21), we find H:

$$H = 46\ \mathrm{km/sec \cdot Mpk} \tag{22}$$

This value of Hubble's constant is very near to that generally acknowledged by astronomers, which is 50 km/sec·Mpk (Klimishin, 1980, 1983).

Hence, formula (20) makes it possible to explain the experimental fact of the slowing down of the revolution of the Moon about the Earth and to calculate the value of Hubble's constant.

From (18) and (22) we find the magnitude of the radius of the space R:

$$R = 4 \times 10^{23}\ \mathrm{km} = 1.3 \times 10^{4}\ \mathrm{Mpk} \tag{23}$$

ACKNOWLEDGMENTS

The author thanks Prof. E. B. Winberg for his discussion of the paper and valuable consultations.

REFERENCES

Abalakin, V. K. (1979). *Osnovy efemeridnoy astronomii*, Nauka, Moscow, p. 337.

Alexeevsky, D. V., Winberg, E. B., and Solodovnikov, A. S. (1988). Geometriya prostranstv postoyannoy Krivizny. *Sovremennye problemy matematiki. Fundamentalnye napravleniya*, VINITI, Moscow, Vol. 29, pp. 5–146.

Duboshin, G. N. (1963). *Nebesnaya mehanika: Metody teorii dvigeniya iskusstvennyh nebesnyh tel*, Nauka, Moscow.

Friedman, A. A. (1979). *Albert Einstein i teoriya gravitatzii*, Mir, Moscow, pp. 330–336.

Gorbatzky, V. G. (1979). *Kosmicheskie vzryvy*, Nauka, Moscow.

Klimishin, I. A. (1980). *Astronomiya nashih dney*, Nauka, Moscow.

Klimishin, I. A. (1983). *Relyativistskaya astronomiya*, Nauka, Moscow.

Landau, L. D., and Lifshitz, E. M. (1973). *Teoriya Polya*, Nauka, Moscow.

Sokolov, A. A., Ternov, I. M., and Gukovsky, V. Ch. (1979). *Kvantovaya mehanika*, Nauka, Moscow, pp. 348–351.

Yefimov, N. V. (1978). *Vysshaya geometriya*, Nauka, Moscow.